

## Simple Tennis MODELS

### 1) Tennis Game MODEL #1

*" You need to hit a ball into the opponent's court, until the opponent misses to hit a ball back to your court. There's no opponent's service, and your service is just hit a ball into opponents court."*

*If an opponent always misses the first ball at 100% of the time, you need to make 1 ball in at least 50% to win a match.*

*If an opponent always misses the 2nd ball at 100% of the time, you need to make 2 balls in at least 50% to win a match, which means that you have to make each ball in with  $(1/2)^{-1/2}$  (71%).*

*If an opponent always misses the 3rd ball at 100% of the time, you need to make 3 balls in at least 50% to win a match, which means that you have to make each ball in with  $(1/2)^{-1/3}$  (79%).*

*Likewise, if an opponent always misses the n-th ball at 100% of the time, you need to make n balls in at least 50% to win a match, which means that you have to make each ball in with an average fraction of  $(1/2)^{-1/n}$ .*

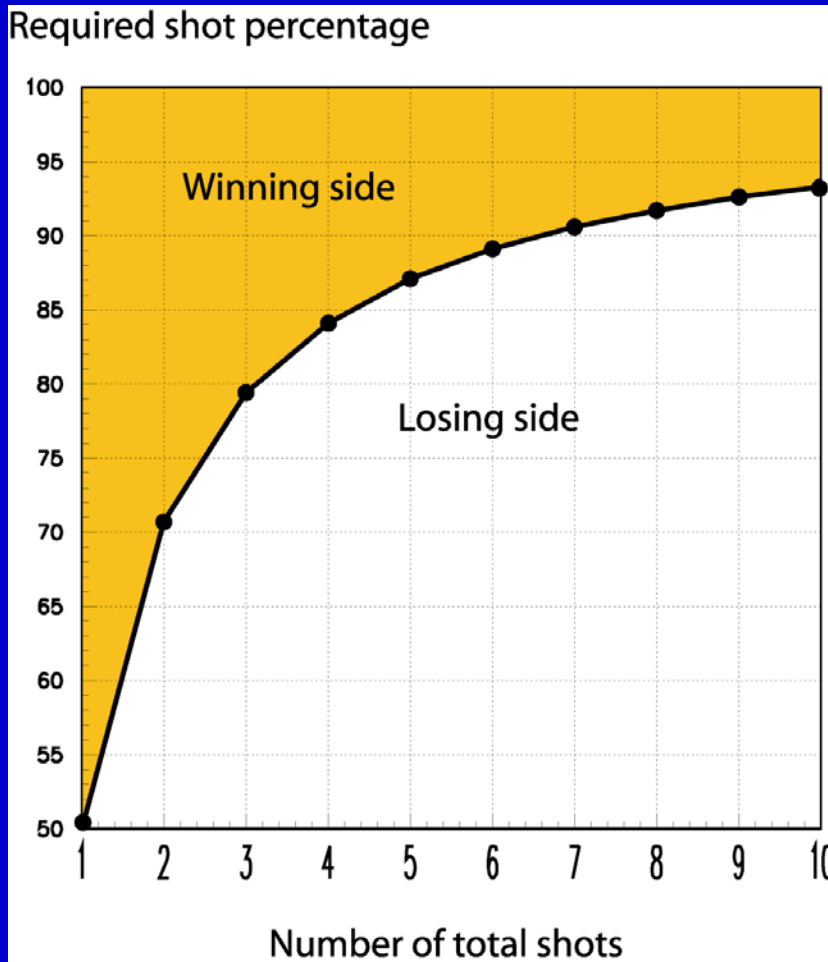


Figure 1 Required shot percentage as a function of number of shots

Figure 1 shows the required shot percentage to win a match, as a function of number of shots to make. If the average shot percentage is below the curve in Figure 1, you'll lose a match.

Opponent's strength(consistency) is shown in how many balls he/she can hit balls back to you. If you have 80% of shot consistency(if you can make 80% of balls in), you can compete with an opponent who can hit 2 balls back. If you can make 90% of balls in, you can compete with an opponent who can hit 6 balls back.

## 2) Tennis Game MODEL #2

*“Suppose a player A and a player B are playing a tennis match, and player A gets a constant fraction,  $X$  ( $0.5 < X < 1.0$ ) to get a point, and player B gets a fraction,  $(1-X)$  to get a point on all points throughout the match”*

*In this tennis game model, the expected probability(Y-axis) of the following can be calculated easily as a function of  $X$  (a player A's point winning fraction):*

- 1) *player A's winning a game* (Figure 2)
- 2) *player A's winning a 6-game set* (Figure 2)
- 3) *player A's winning a best 3 set match,* (Figure 2)
- 4) *player A's winning a deuce game* (Figure 3)
- 5) *player A's winning a 7 point tie-breaker* (Figure 3)
- 6) *Player A's winning a best 3 set with 2 straight sets* (Figure 4)
- 7) *Player A's winning a best 3 set with 3 sets* (Figure 4)
- 8) *Player B's winning a best 3 set with 2 straight sets* (Figure 4)
- 9) *Player B's winning a best 3 set with 3 sets* (Figure 4)
- 10) *Player A's winning a game with love* (Figure 5)
- 11) *Player A's winning a game with 15* (Figure 5)
- 12) *Player A's winning a game with 30* (Figure 5)
- 13) *Player A's winning a game with Deuce* (Figure 5)
- 14) *Player A's winning a set with 6-0* (Figure 6)
- 15) *Player A's winning a set with 6-10* (Figure 6)
- 16) *Player A's winning a set with 6-2* (Figure 6)
- 17) *Player A's winning a set with 6-3* (Figure 6)
- 18) *Player A's winning a set with 6-4* (Figure 6)
- 19) *Player A's winning a set with 7-5* (Figure 6)
- 20) *Player A's winning a set with a tie-breaker* (Figure 6)

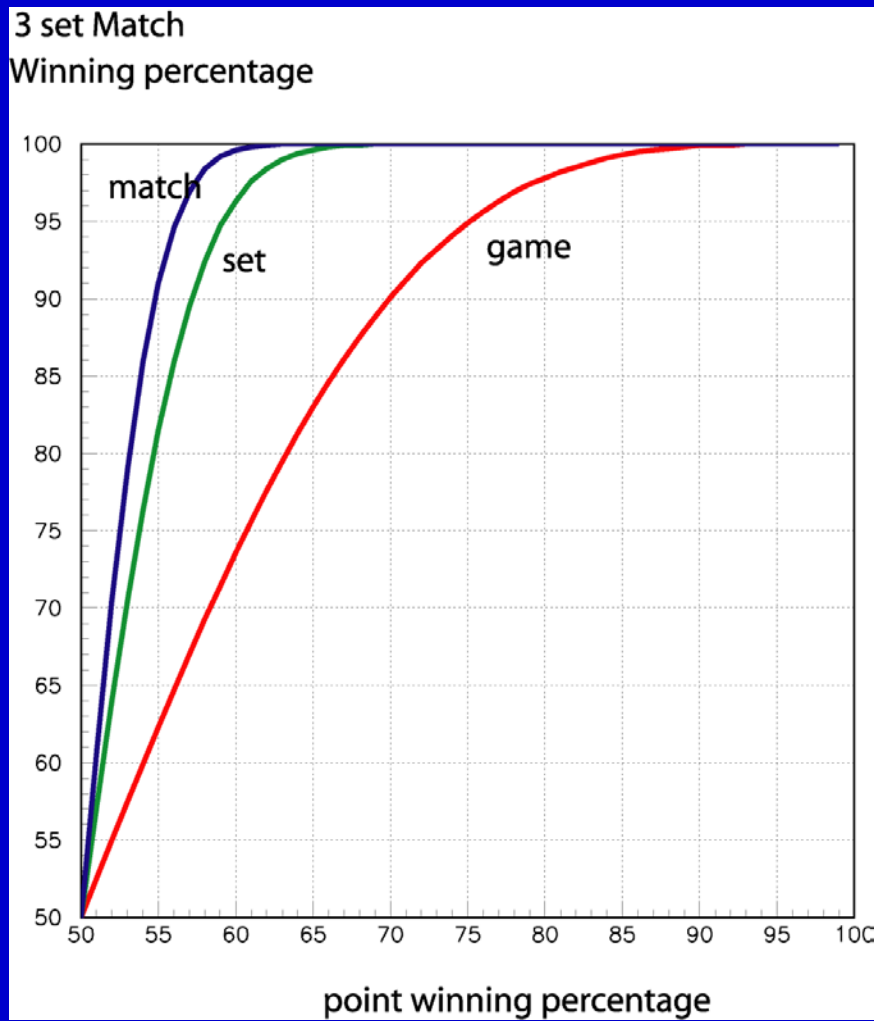


Figure 2 Winning percentage as a function of point winning percentage

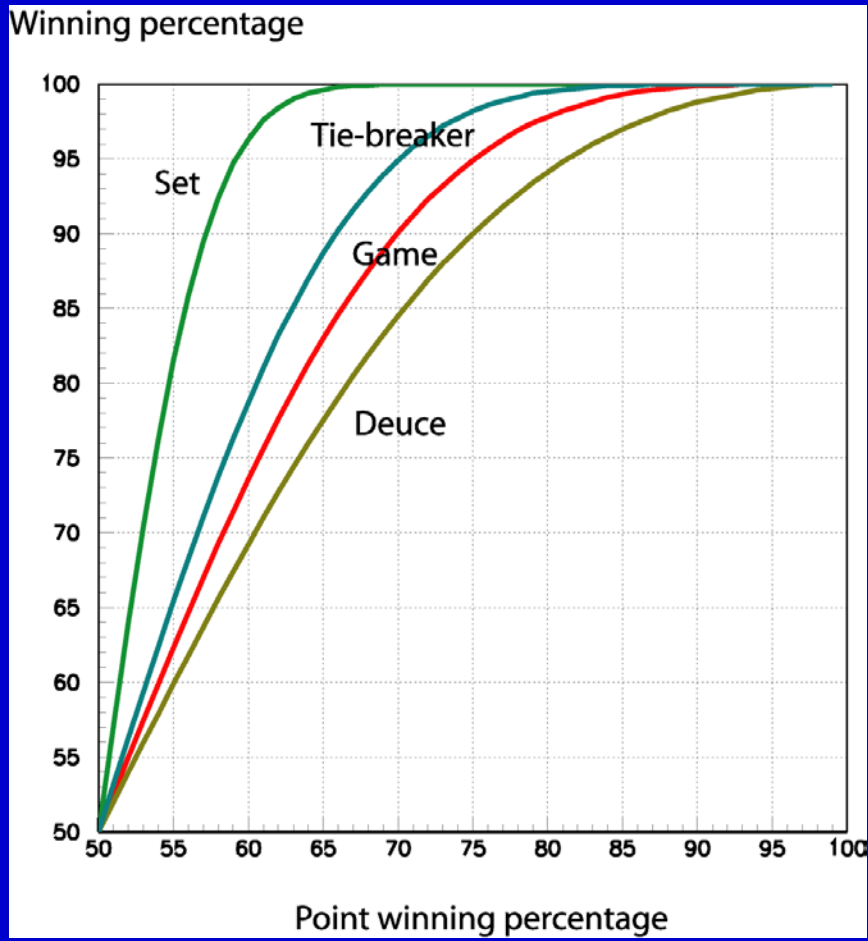


Figure 3 Winning percentage as a function of point winning percentage

### Winning percentage

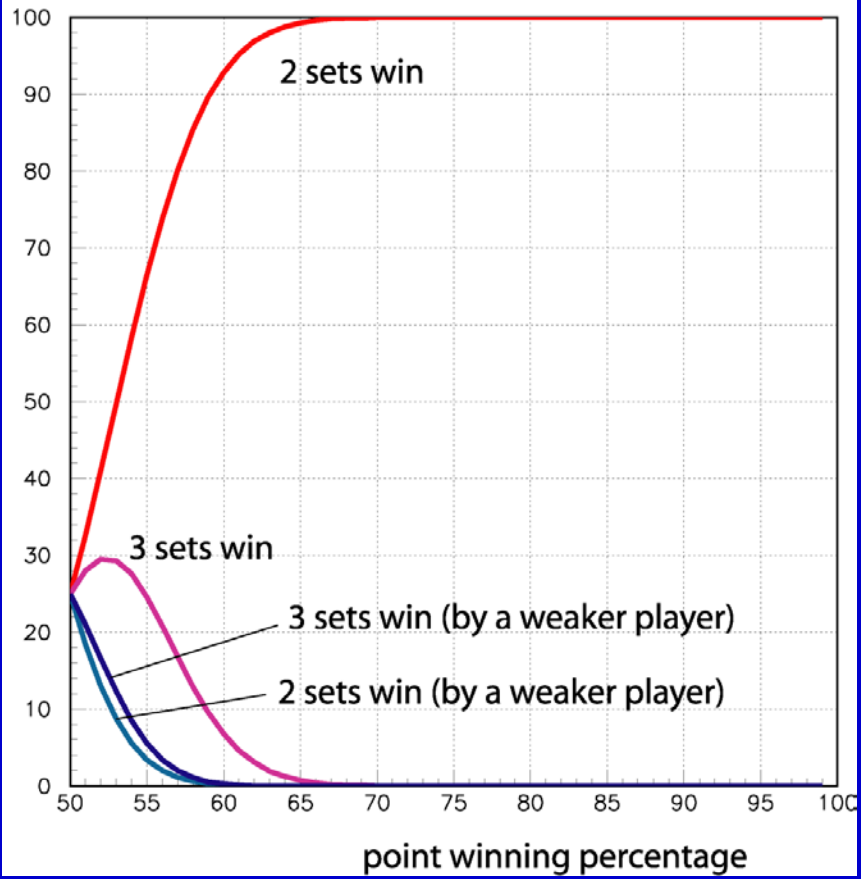


Figure 4 Winning percentage as a function of point winning percentage

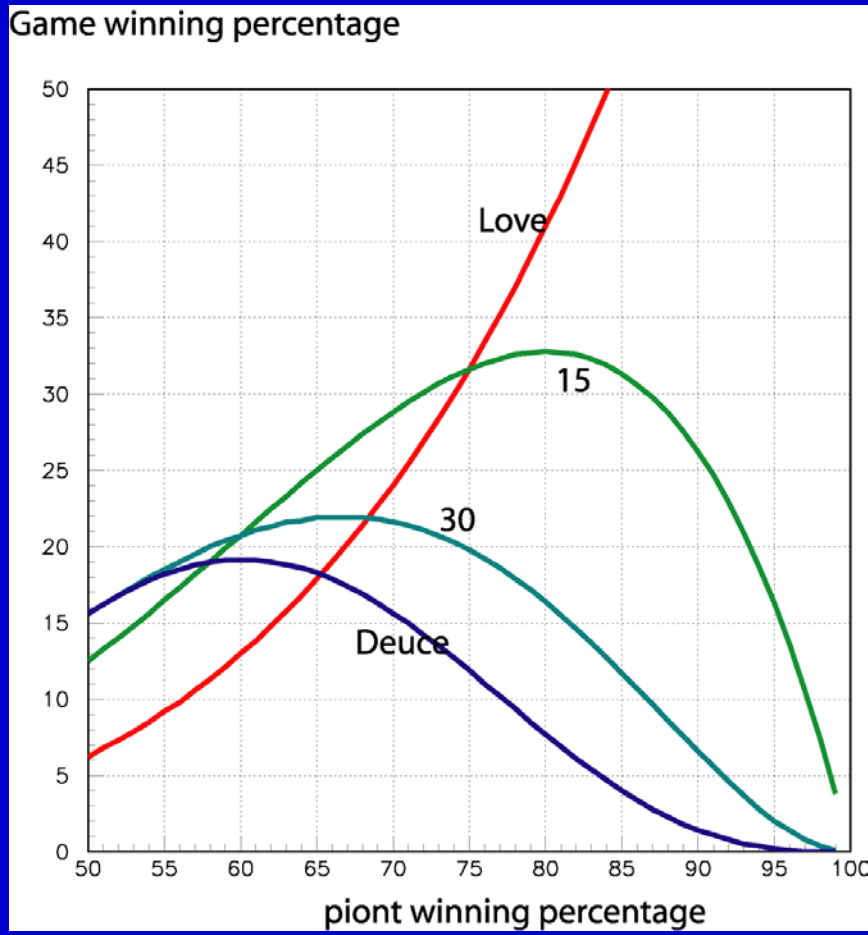


Figure 5 Game winning percentage as a function of point winning percentage

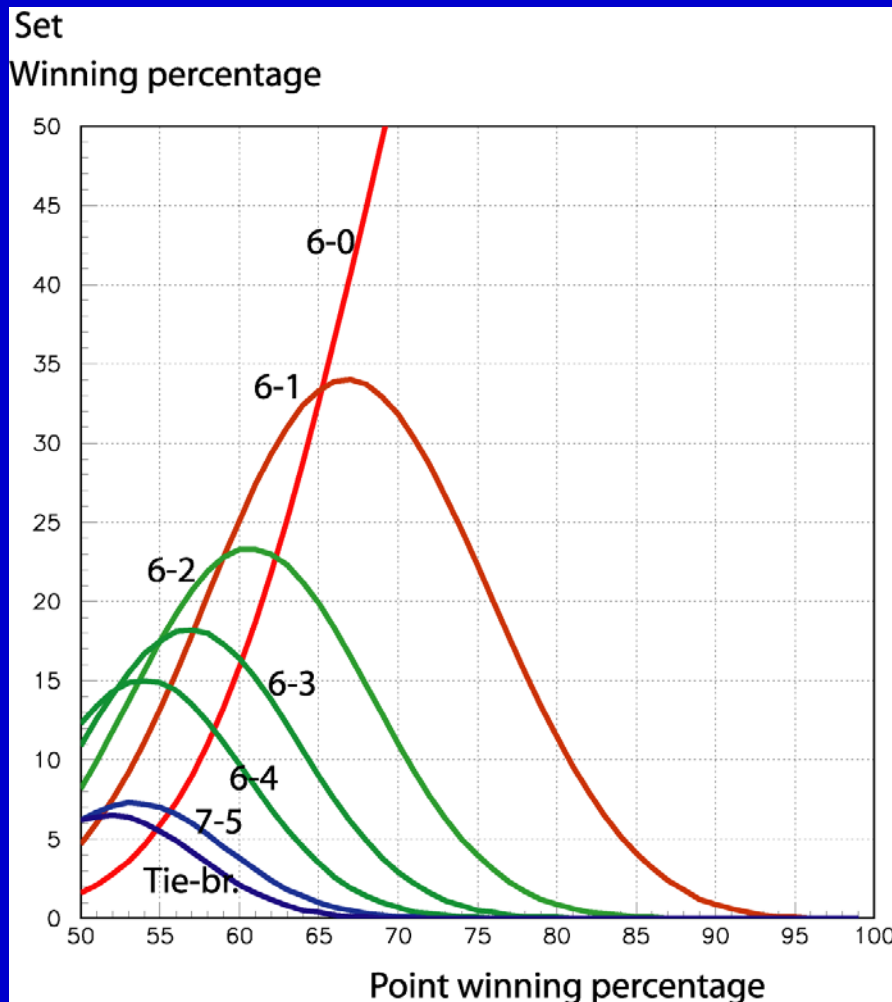


Figure 6 Set winning percentage as a function of point winning percentage

If a player A gets 60% of points, Player A wins 74% of games (69% in a deuce), 96% of sets (78% in a tie-breaker), and 99.6% of matches against a player B with 40% of point winning percentage. At the same percentage, player B with 40% of point winning percentage wins 26% of games (31% in a deuce), 4% of sets (22% in a tie-breaker), and 0.4% of matches. Player A has 2 sets win 93% of time and 3 sets win 7% of time. Player A with 60% point winning percentage win a game by mostly 15 or 30 in a game, followed by deuce games, and the player A wins a set mostly with 6-1 or 6-2, for example with the point winning percentage. A weaker player (with lower point winning percentage) has more chances to win in a deuce game (so called "Break (in) Deuce"), in a tie-breaker, and in a 3 set matches (Figures 3 and 4).



Interestingly, at 50-50 point (EQUAL) winning percentages of players A and B, most games are 30-games or deuce games, and set counts are mostly 6-4 or 6-3 (Figures 5 and 6).

For example, let  $a$ , and  $b$  represent events that player A or player B get a point in a match. Any tennis match is then represented completely by a sequence of  $a$  or  $b$ .

If a player gets 4 points with 2 or more points difference from the sum of opponent's points, a player gets a game. If both a player and an opponent gets 3 points in a game, the game is called as a deuce game. 0 point is called Love, 1 point is called as 15, 2 points are called as 30, and 4 points are called as 40, in a game.

If a player gets 6 games with 2 or more games difference from the sum of opponent's games, a player gets a set. If both a player and an opponent gets 6 games in a set, a tie-breaker game is played, till a side gets 7 points with 2 or more points difference.

The best of 3 set match is won by a player by taking 2 sets.

If a player A gets a love-game, which is 4 points in a row in a game, the probability to have the case  $aaaa$  happen is:

$X^4$ , which is shown in "Love" curve in Figure 4.

If a player B gets 2 points (30) in A's winning game, 2 b's and 3 a's must happen before A's getting the 6<sup>th</sup> point. An example is  $abaaba$ . The number of cases for arranging  $a$  and  $b$  in that way is  ${}_5C_2 = (5 \times 4) / 2 = 8$ . So the probability of A's 30-game is

$8 \times X^4 (1-X)^2$ , which is shown in "30" curve in Figure 4.

A deuce game happens if  $ab$  or  $ba$  follows after 30-all. Probability to have this happen is  ${}_2C_1 X^4 C_2 X^3 (1-X)^3$ . For a player A to win a deuce game, the following have to follow:  $aa$ , or  $(ab \text{ or } ba)aa$ ,  $(ab \text{ or } ba)(ab \text{ or } ba)aa$ , ... , which probability is:  $X^2 + {}_2C_1 X(1-X)X^2 + \{ {}_2C_1 X(1-X) \}^2 X^2 + \dots = X^2 / \{ 1 - X(1-X) \}$ . So the A's winning probability in a deuce game is:

${}_2C_1 \cdot {}_4C_2 X^3 (1-X)^3 * X^2 / \{ 1 - X(1-X) \} = 12 X^5 (1-X)^3 / \{ 1 - X(1-X) \}$ .

Other winning probability is calculated in the same way. Please try to figure out the other probability functions by your self.

Reference: "The science of Tennis" by K. Miura and T. Chomabayashi, Koubunsha Publishing Company, Japan, 1980.

